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# UNSTEADY MHD FLOW DUE TO NON-COAXIAL ROTATIONS OF A POROUS DISK AND A FLUID AT INFINITY SUBJECTED TO A PERIODIC SUCTION

M. GURIA

Department of Mathematics, Ghatal Rabindra Satabarsiki Mahavidyalaya Ghatal, Paschim Medinipur, West Bengal, INDIA E-mail: mrinmoy9832@yahoo.com

The unsteady flow of a viscous incompressible electrically conducting fluid due to non-coaxial rotations of a porous disk subjected to a periodic suction and the fluid at infinity in the presence of applied transverse magnetic field has been studied. The fluid at infinity passes through a fixed point. The velocity field, shear stresses are obtained in a closed form.

Key words: Hall currents, eccentric, mhd, oscillating, rotating.

### 1. Introduction

The flow of a viscous incompressible fluid due to non-coaxial rotations of a disk and the fluid at infinity has been studied by a number of researchers. An exact solution of this type of problem was obtained by Berker [1]. Coirier [2] studied the flow due to a disk and the fluid at infinity which are rotating noncoaxially at a slightly different angular velocity. The non-Newtonian flow due to a disk and the fluid at infinity which are rotating non-coaxially at a slightly different angular velocity was studied by Erdogan [3]. An exact solution of the three dimensional Navier-Stokes equations for the flow due to non coaxial rotation of a porous disk and the fluid at infinity was studied by Erdogan [4, 5]. Murthy and Ram [6] studied the magnetohydrodynamic flow and heat transfer due to eccentric rotations of a porous disk and the fluid at infinity. The unsteady flow due to non-coaxial rotations of a disk, oscillating in its own plane and the fluid at infinity was studied by Kasiviswanathan and Rao [7]. Chakraborti et al. [8] studied the hydromagnetic flow due to non-coaxial rotations of a disk and the fluid at infinity with same angular velocity. The flow due to non-coaxial rotations of an oscillatory porous disk and the fluid at infinity about an axis passing through a fixed point parallel to the axis of rotation of the disk was investigated by Hayet et al. [9]. The flow due to non-coaxial rotations of an oscillating porous disk and the fluid at infinity which rotate about an axis passing through a fixed point parallel to the axis of rotation of the disk was studied by Guria et al. [10]. Hayet et al. [11] studied the unsteady MHD flow due to non-coaxial rotations of a porous disk and the fluid at infinity.

Hayet *et al.* [12] also investigated the peridic MHD flow due to non coaxial rotations of a porous disk and the fluid at infinity. Recently Ghosh *et al.* [13] studied the flow due to non coaxial rotations of a porous disk subjected to a periodic suction and the fluid at infinity. In the present paper, we have studied the MHD flow due to non coaxial rotations of a porous disk subjected to a periodic suction of a porous disk subjected to a periodic suction and the fluid at infinity. In the present paper, we have studied the fluid at infinity in the presence of an applied tranverse magnetic field.

## 2. Formulation of the problem and its solution

Consider the unsteady flow of a viscous incompressible conducting fluid due to the rotation of a porous disk rotating about the *z*-axis with uniform angular velocity  $\Omega$ . The fluid at infinity  $(z \rightarrow \infty)$  rotates about an axis parallel to the *z*-axis passing through the point  $(x_I^*, y_I^*)$  with the same angular velocity  $\Omega$ . A uniform magnetic field  $B_0$  is imposed parallel to the *z*-axis [See Fig.1]. Due to the periodic suction the flow becomes three-dimensional.



Fig.1. Geometry of the problem.

We assume the velocity components of the form

$$u = -\Omega y + f(z,t), \quad v = \Omega x + g(z,t), \quad w = -V(t).$$
 (2.1)

The boundary conditions of the problem are

$$u = -\Omega y, \quad v = \Omega x, \quad w = -V(t) \quad \text{at} \quad z = 0,$$
  
$$u = -\Omega(y - y_1), \quad v = \Omega(x - x_1), \quad w = -V(t) \quad \text{as} \quad z \to \infty.$$
  
(2.2)

The Navier-Stokes equations of motion are

$$\nabla \boldsymbol{.} \boldsymbol{q} = \boldsymbol{0} , \qquad (2.3)$$

$$\frac{\partial \boldsymbol{q}}{\partial t} + (\boldsymbol{q}.\nabla)\boldsymbol{q} = -\frac{l}{\rho}\nabla \boldsymbol{p} + \nu\nabla^2 \boldsymbol{q} + \frac{l}{\rho}(\boldsymbol{J} \times \boldsymbol{B}), \qquad (2.4)$$

$$\frac{l}{\rho} \left( \boldsymbol{J} \times \boldsymbol{B} \right) = -\frac{\sigma}{\rho} B_0^2 V.$$
(2.5)

Introducing Eq.(2.1), the Navier Stokes equations become respectively

$$-\frac{l}{\rho}\frac{\partial p}{\partial x} = \Omega^2 x + \Omega g + \frac{\partial f}{\partial z} + \frac{\partial f}{\partial t} + v \frac{\partial^2 f}{\partial z^2} - \frac{\sigma}{\rho}B_0^2 \Big[f(z,t) - \Omega y\Big],$$
(2.6)

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = \Omega^2 y - \Omega f + V(t)\frac{\partial g}{\partial z} - \frac{\partial g}{\partial t} + v\frac{\partial^2 g}{\partial z^2} - \frac{\sigma}{\rho}B_0^2 \left[g(z,t) + \Omega x\right]$$
(2.7)

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{\sigma}{\rho}B_0^2 V(t) + \frac{\partial V(t)}{\partial t}$$
(2.8)

where (u,v,w) are the velocity components along the coordinate axes,  $\rho$  is the fluid density, v is the kinematic coefficient of viscosity.  $\frac{\partial p}{\partial z}$  is a function of t only, if we differentiate with respect to x and y then it is zero.

The perodic suction velocity distibution of the form

$$V(t) = -W_0 \left[ 1 + \varepsilon A e^{i\sigma t} \right]$$
(2.9)

where  $w_0$  is a positive constant,  $\varepsilon > 0$  is very small and A is a real positive constant such that  $\varepsilon A \le I$ . Differentiating Eqs (2.6) and (2.7), we get

$$\Omega \frac{\partial g}{\partial z} + V(t) \frac{\partial^2 f}{\partial z^2} - \frac{\partial^2 f}{\partial z \partial t} + v \frac{\partial^3 f}{\partial z^3} - \frac{\sigma}{\rho} B_0^2 \frac{\partial f}{\partial z} = 0, \qquad (2.10)$$

$$-\Omega \frac{\partial f}{\partial z} + V(t) \frac{\partial^2 g}{\partial z^2} - \frac{\partial^2 g}{\partial z \partial t} + v \frac{\partial^3 g}{\partial z^3} - \frac{\sigma}{\rho} B_0^2 \frac{\partial g}{\partial z} = 0, \qquad (2.11)$$

Combining Eqs (2.10) and (2.11), we get

$$\nu \frac{\partial^3 F}{\partial z^3} + V(t) \frac{\partial^2 F}{\partial z^2} - \frac{\partial^2 F}{\partial z \partial t} - \left(i\Omega + \frac{\sigma}{\rho}B_0^2\right) \frac{\partial F}{\partial z} = 0$$
(2.12)

where

$$F = f + ig, \qquad i = \sqrt{-I}. \tag{2.13}$$

The corresponding boundary conditions become

$$F(0,t) = 0, \qquad F(\infty,t) = -i\Omega(x_1 + iy_1). \tag{2.14}$$

We assume

$$F(z,t) = F_0(z,t) + \varepsilon F_1(z,t) e^{i\sigma t} .$$
(2.15)

Substituitng (2.15) in Eq.(2.12) and comparing the harmonics terms and neglecting the coefficients of  $\epsilon^2$  .

$$\nu \frac{d^3 F_0}{dz^3} + w_0 \frac{d^2 F_0}{\partial z^2} - \left(i\Omega + \frac{\sigma}{\rho} B_0^2\right) \frac{dF_0}{\partial z} = 0, \qquad (2.16)$$

$$v\frac{d^{3}F_{I}}{dz^{3}} + w_{0}\frac{d^{2}F_{I}}{\partial z^{2}} - \left(i\Omega + i\sigma + \frac{\sigma}{\rho}B_{0}^{2}\right)\frac{dF_{I}}{\partial z} = -w_{0}A\frac{d^{2}F_{0}}{dz^{2}}.$$
(2.17)

Introducing the non-dimensional variables

$$\eta = \sqrt{\frac{\Omega}{2\nu}}z, \qquad S = \frac{w_0}{2\sqrt{\Omega\nu}}, \qquad n = \sqrt{I + \frac{\sigma}{\Omega}}, \qquad M^2 = \frac{2\sigma B_0^2}{\rho\Omega}, \tag{2.18}$$

Eqs (2.16) and (2.17) become

$$\frac{d^3 F_0}{d\eta^3} + 2\sqrt{2}S \frac{d^2 F_0}{\partial \eta^2} - \left(2i + M^2\right) \frac{dF_0}{\partial \eta} = 0, \qquad (2.19)$$

$$\frac{d^{3}F_{I}}{dz^{3}} + 2\sqrt{2}S\frac{d^{2}F_{I}}{\partial\eta^{2}} - \left(2i\eta^{2} + M^{2}\right)\frac{dF_{I}}{\partial\eta} = -2\sqrt{2}SA\frac{d^{2}F_{0}}{d\eta^{2}}.$$
(2.20)

The boundary conditions (2.14) become

$$F_0(0) = 0, \quad F_0(\infty) = -i\Omega(x_1 + iy_1), \quad F_1(0) = 0, \quad F_1(\infty) = 0.$$
(2.21)

Solving Eqs (2.19) and (2.20) subject to the boundary conditions (2.21) and using Eq.(2.13), we get

$$\frac{f}{\Omega} = y_I \left( I - e^{-\alpha_0 \eta} \cos \beta_0 \eta \right) + x_I e^{-\alpha_0 \eta} \sin \beta_0 \eta + 
+ \frac{\varepsilon \sqrt{2}SA}{\left(n^2 - I\right)} \left[ \left( \alpha_0 x_I - \beta_I y_I \right) \left\{ e^{-\alpha_I \eta} \cos \left( \beta_I \eta - \sigma t \right) - e^{-\alpha_0 \eta} \cos \left( \beta_0 \eta - \sigma t \right) \right\} + 
+ \left( \alpha_0 y_I + \beta_0 x_I \right) \left\{ e^{-\alpha_I \eta} \sin \left( \beta_I \eta - \sigma t \right) - e^{-\alpha_0 \eta} \sin \left( \beta_0 \eta - \sigma t \right) \right\} \right],$$
(2.22)

$$\frac{g}{\Omega} = y_I e^{-\alpha_0 \eta} \sin \beta_0 \eta - x_I (I - e^{-\alpha_0 \eta}) \cos \beta_0 \eta + + \frac{\varepsilon \sqrt{2}SA}{(n^2 - I)} \Big[ (\alpha_0 y_I + \beta_0 x_I) \Big\{ e^{-\alpha_I \eta} \cos(\beta_I \eta - \sigma t) - e^{-\alpha_0 \eta} \cos(\beta_0 \eta - \sigma t) \Big\} + + (\alpha_0 x_I - \beta_0 y_I) \Big\{ e^{-\alpha_I \eta} \sin(\beta_I \eta - \sigma t) - e^{-\alpha_0 \eta} \sin(\beta_0 \eta - \sigma t) \Big\} \Big]$$
(2.23)

where

$$\begin{aligned} \alpha_{0} &= \sqrt{2}S + \gamma_{0}, \qquad \alpha_{I} = \sqrt{2}S + \gamma_{I} \\ \gamma_{0} &= \left\{ \frac{2S^{2} + M^{2} + \sqrt{\left(2S^{2} + M^{2}\right)^{2} + 4}}{4} \right\}^{1/2}, \\ \beta_{0} &= \left\{ \frac{-\left(2S^{2} + M^{2}\right) + \sqrt{\left(2S^{2} + M^{2}\right)^{2} + 4}}{4} \right\}^{1/2}, \\ \gamma_{I} &= \left\{ \frac{2S^{2} + M^{2} + \sqrt{\left(2S^{2} + M^{2}\right)^{2} + 4n^{4}}}{4} \right\}^{1/2}, \\ \beta_{I} &= \left\{ \frac{-\left(2S^{2} + M^{2}\right) + \sqrt{\left(2S^{2} + M^{2}\right)^{2} + 4n^{4}}}{4} \right\}^{1/2}. \end{aligned}$$

$$(2.24)$$

The effect of variable suction introduces a transient part depending on  $\varepsilon$ , A and  $\sigma$  superposed on the steady solution corresponding to uniform suction at the disk. If S = 0, the problem recovers the solution for the steady Ekman layer on the disk.

**Case.I:** When A=0

$$\frac{f}{\Omega} = y_I \left( I - e^{-\alpha_0 \eta} \cos \beta_0 \eta \right) + x_I e^{-\alpha_0 \eta} \sin \beta_0 \eta$$
$$\frac{g}{\Omega} = y_I e^{-\alpha_0 \eta} \sin \beta_0 \eta - x_I (I - e^{-\alpha_0 \eta}) \cos \beta_0 \eta$$

**Case.II:** When  $x_l = 0$ ,  $y_l = l$ 

$$\frac{f}{\Omega l} = \left(l - e^{-\alpha_0 \eta} \cos \beta_0 \eta\right) + \\
+ \frac{\varepsilon \sqrt{2} SA}{\left(n^2 - l\right)} \left[ -\beta_0 \left\{ e^{-\alpha_1 \eta} \cos(\beta_1 \eta - \sigma t) - e^{-\alpha_0 \eta} \cos(\beta_0 \eta - \sigma t) \right\} + \\
+ \alpha_0 \left\{ e^{-\alpha_1 \eta} \sin(\beta_1 \eta - \sigma t) - e^{-\alpha_0 \eta} \sin(\beta_0 \eta - \sigma t) \right\} \right],$$
(2.25)

$$\frac{g}{\Omega l} = e^{-\alpha_0 \eta} \sin \beta_0 \eta + \\
+ \frac{\varepsilon \sqrt{2} S A}{\left(n^2 - l\right)} \left[ \left( \alpha_0 y_l + \beta_0 x_l \right) \left\{ e^{-\alpha_l \eta} \cos \left( \beta_l \eta - \sigma t \right) - e^{-\alpha_0 \eta} \cos \left( \beta_0 \eta - \sigma t \right) \right\} + \\
+ \left( \alpha_0 x_l - \beta_0 y_l \right) \left\{ e^{-\alpha_l \eta} \sin \left( \beta_l \eta - \sigma t \right) - e^{-\alpha_0 \eta} \sin \left( \beta_0 \eta - \sigma t \right) \right\}.$$
(2.26)

**Case.III:** When A = 0,  $M^2 = 0$ , the results (2.22) and (2.23) are the same as reference [7].

**Case.IV:** When  $x_1 = 0$ ,  $y_1 = l$ , A = 0,  $M^2 = 0$  and -f by g and g by f, the results are the same as reference [4].

**Case.V:** If A = 0, the problem is reduced to Hayet *et al.* [12].

The flow very near the porous disk

$$\frac{f}{\Omega l} = \alpha_0 \eta + \frac{\varepsilon \sqrt{2}SA}{\left(n^2 - l\right)} \Big[ (\alpha_1 - \alpha_0) \beta_0 \eta \cos \sigma t - (\beta_1 - \beta_0) \beta_0 \eta \sin \sigma t + (\beta_1 - \beta_0) \alpha_0 \eta \cos \sigma t - (\alpha_1 - \alpha_0) \beta_0 \eta \sin \sigma t \Big],$$
(2.27)

$$\frac{g}{\Omega l} = \beta_0 \eta + \frac{\varepsilon \sqrt{2} SA}{\left(n^2 - l\right)} \Big[ -\left(\alpha_l - \alpha_0\right) \alpha_0 \eta \cos \sigma t + \left(\beta_l - \beta_0\right) \beta_0 \eta \sin \sigma t + \left(\alpha_l - \alpha_0\right) \beta_0 \eta \sin \sigma t + \left(\beta_l - \beta_0\right) \beta_0 \eta \sin \sigma t \Big].$$
(2.28)

The inclination of the fluid velocity vector with the y axis near z = 0 becomes  $\theta = \tan^{-1} (C/D)$ , where

$$C = \beta_0 \left( n^2 - I \right) + \varepsilon \sqrt{2} SA \left[ \left( \alpha_1 - \alpha_0 \right) \beta_0 \eta \cos \sigma t - \left( \beta_1 - \beta_0 \right) \beta_0 \eta \sin \sigma t + \left( \beta_1 - \beta_0 \right) \alpha_0 \eta \cos \sigma t - \left( \alpha_1 - \alpha_0 \right) \beta_0 \eta \sin \sigma t \right],$$

$$D = \alpha_0 \left( \eta^2 - I \right) + \varepsilon \sqrt{2} SA \left( n^2 - I \right) \left[ \left( \alpha_1 - \alpha_0 \right) \beta_0 \eta \cos \sigma t + \left( \beta_1 - \beta_0 \right) \beta_0 \eta \sin \sigma t + \left( \beta_1 - \beta_0 \right) \alpha_0 \eta \cos \sigma t - \left( \alpha_1 - \alpha_0 \right) \beta_0 \eta \sin \sigma t \right].$$

If  $\sigma t = \pi / 2$ ,  $S \neq 0$ ,  $A \neq 0$ , the inclination of the fluid velocity to the y axis near the z axis will be

$$\theta = \tan^{-I} \left\{ \frac{\beta_0 \left( n^2 - I \right) + \varepsilon \sqrt{2} SA \left[ \left( \beta_I - \beta_0 \right) \alpha_0 + \left( \alpha_I - \alpha_0 \right) \beta_0 \right]}{\alpha_0 \left( n^2 - I \right) + \varepsilon \sqrt{2} SA \left[ \left( \alpha_I - \alpha_0 \right) \alpha_0 - \left( \beta_I - \beta_0 \right) \beta_0 \right]} \right\}$$
(2.29)

#### 3. Results and discussion

I have presented the non-dimensional primary velocity  $f/\Omega l$  and the secondary velocity  $g/\Omega l$  against  $\eta$  for several values of  $\sigma t$ , the magnitude of fluctuation of suction velocity A, the suction or blowing parameter S, the magnetic parameter  $M^2$ . Figure 2 shows the variations of the primary velocity for

several values of  $\sigma t$ . It is observed that the primary velocity decreases with an increase in  $\sigma t$  near the disk and it is almost stationary away from the disk. The reverse effect is observed for secondary velocity. In Fig.3 it is observed that both the primary and secondary velocities increase with an increase in A. Figure 4 shows that the effect of suction or blowing parameter. It is seen that suction creates thinning of the boundary layer. The crossing of the graphs shown in the figure is due to the presence of suction because suction result in thinning of the boundary layer near the disk and thickening of the boundary layer away from the disk. Figure 5 represents the variations of  $f/\Omega l$  and the secondary velocity  $g/\Omega l$  for several values of the magnetic

parameter  $M^2$ . It is found that  $f/\Omega l$  increases but  $g/\Omega l$  decreases with an increase in magnetic parameter.



Fig.2. Variation of primary velocity for S = 1, A = 2,  $M^2 = 10$ , t = 0.1,  $n^2 = 1.5$ ,  $x_1 = 0$ ,  $y_1 = 1$ .



Fig.3: Variation of primary velocity for S = 1,  $M^2 = 10$ ,  $\sigma t = \pi / 4$ , t = 0.1,  $n^2 = 1.5$ ,  $x_1 = 0$ ,  $y_1 = 1$ .



Fig.4. Variation of primary velocity for  $M^2 = 10$ ,  $\sigma t = \pi / 4$ , t = 0.1,  $n^2 = 1.5$ ,  $x_1 = 0$ ,  $y_1 = 1$ .



Fig.5. Variation of primary velocity for S = 1,  $\sigma t = \pi / 4$ , t = 0.1,  $n^2 = 1.5$ ,  $x_1 = 0$ ,  $y_1 = 1$ .



Fig.6. Variation of primary velocity for S = 1,  $\sigma t = \pi / 4$ , t = 0.1,  $n^2 = 1.5$ .

Using Eq.(2.22) and Eqs (2.35) and (2.36), we get the non-dimensional shear stress components  $\tau_x$  and  $\tau_y$  at the disk as

$$\frac{\partial}{\partial \eta} \left( \frac{f}{\Omega y_I} \right) = \left( \alpha_0 + \beta_0 \frac{x_I}{y_I} \right) + \frac{\varepsilon \sqrt{2} SA}{\left(n^2 - I\right)} \left[ \left( \alpha_0 \frac{x_I}{y_I} - \beta_0 \right) \left\{ (\beta_I - \beta_0) \sin \sigma t - (\alpha_I - \alpha_0) \cos \sigma t \right\} + \left( \alpha_0 + \beta_0 \frac{x_I}{y_I} \right) \left\{ (\alpha_I - \alpha_0) \sin \sigma t + (\beta_I - \beta_0) \cos \sigma t \right\} \right],$$

$$(3.1)$$

$$\frac{\partial}{\partial \eta} \left( \frac{g}{\Omega y_I} \right) = \left( \beta_0 - \alpha_0 \frac{x_I}{y_I} \right) + \frac{\varepsilon \sqrt{2} SA}{\left(n^2 - I\right)} \left[ \left( \alpha_0 + \beta_0 \frac{x_I}{y_I} \right) \left\{ (\beta_I - \beta_0) \sin \sigma t - (\alpha_I - \alpha_0) \cos \sigma t \right\} + \left( \alpha_0 \frac{x_I}{y_I} - \beta_0 \right) \left\{ (\alpha_I - \alpha_0) \sin \sigma t + (\beta_I - \beta_0) \cos \sigma t \right\} \right].$$

$$(3.2)$$

The non-dimensional shear stresses  $\tau_x$  and  $\tau_y$  due to the primary and the secondary flows at the disk are entered in Tab.1 for different values of *m* and  $\omega$  with  $S = I, \sigma = 5, \sigma \tau = \frac{\pi}{2}, x_I = y_I = I, M^2 = 5$ .

	$\tau_x$			$-\tau_y$	$-\tau_y$		
$S \setminus M^2$	5	10	15	5	10	15	
-2.0	0.88	1.38	2.28	0.55	1.21	2.28	
-1	1.44	2.11	3.21	0.90	1.78	3.07	
0	2.71	3.49	4.70	1.83	2.86	4.25	
1	5.06	5.73	6.84	3.76	4.62	5.90	
2	8.13	8.64	9.53	6.24	6.88	7.96	

Table 1. Shear stresses due to the primary and secondary flows for  $M^2 = 5$ .

It is observed that the shear stresses  $\tau_x$  and magnitude of  $\tau_y$  increase with an increase in both suction parameter and magnetic parameter.

## Conclusion

The unsteady flow of a viscous incompressible fluid due to non-coaxial rotations of a porous disk and the fluid at infinity subjected to a periodic suction in the presence of a magnetic field has been studied. It is seen that suction creates thinning of the boundary layer. The crossing of the graphs shown in the figure is due to the presence of suction because suction creates thinning of the boundary layer near the disk and thickening of the boundary layer away from the disk. It is found that the primary velocity increases but secondary velocity decreases with an increase in the magnetic parameter.

## Nomenclature

- A amplitude of the suction velocity
- B magnetic field vector
- $B_0$  applied magnetic field
- C,D constants

 $f / \Omega l, g / \Omega l$  – dimensionless velocity components in the x, z -axes, respectively

- J current density
- l distance between the axes of rotation
- $M^2$  magnetic parameter
  - n rotational parameter
  - p pressure
  - S constant suction velocity
- u, v, w velocity components in the x, y, z -axes respectively
  - V suction velocity
  - $w_0$  constant
  - x,y,z Cartesian coordinate system
    - $\Omega$  angular velocity
    - $\eta$  dimensionless *z* coordinate system
    - $\nu$  kinematic viscosity
    - $\rho~$  density of the fluid
    - $\epsilon$  constant
- $\tau_x, \tau_z$  shear stress due to primary and secondary flows
- $\alpha_0, \alpha_1$  constants
- $\beta_0, \beta_1$  constants
- $\gamma_0, \gamma_1$  constants
  - $\theta$  angle

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